BLIND SOURCE SEPARATION FOR SOME MULTIVARIATE HETEROSCEDASTIC TIME SERIES MODELS

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Blind Source Seperation (BSS) model is a semiparametric model, where the components of a p-variate vector $\boldsymbol{x}$ are assumed to be a linear combinations of the components of some unobserved p-variate source vector $\boldsymbol{z}$. In time series context observations $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T$ are from a p-variate time series and $\boldsymbol{\Omega}$ is a full-rank $p \times p$ mixing matrix. We assume that these observations obey a BSS model $\boldsymbol{x}_t = \boldsymbol{\Omega} \boldsymbol{z}_t$ ($t = 0, \pm 1, \pm 2, \ldots$), where $(\boldsymbol{z}_t)_{0, \pm 1, \pm 2, \ldots}$ is a p-variate series.

In independent component analysis (ICA) the components of $\boldsymbol{z}_t$ are mutually independent. In FOBI approach one uses the fact that $E[\boldsymbol{z}_t \boldsymbol{z}_t' \boldsymbol{z}_t \boldsymbol{z}_t'] = \boldsymbol{D}$ is a diagonal matrix, where the diagonal elements are in decreasing order. In FOBI the diagonal elements are assumed to be distinct.

In second order blind source separation (SOS) models the p time series in $\boldsymbol{z}_t$ are assumed to be weakly stationary and uncorrelated. The solutions for SOS models can be found using e.g. AMUSE estimator or SOBI method. In SOBI (Second Order Blind Identification) the idea is to use several autocovariance matrices and jointly diagonalize them to get an unmixing matrix. However, SOBI and FOBI do not normally work in the case of heteroscedastic time series.

In our new fourth order blind source separation (FOS) model for heteroscedastic time series models we assume that $E[\boldsymbol{z}_{t+k} \boldsymbol{z}_t' \boldsymbol{z}_t \boldsymbol{z}_{t+k}'] = \boldsymbol{D}_k$, where $\boldsymbol{D}_k$ is a diagonal matrix, where all the diagonal elements are distinct, and the lag $k = 1, 2, \ldots$. We have introduced a new algorithm for this situation.

In simulation study, using diffent multivariate heteroscedastic processes, we have shown the advantage of our method over FOBI and SOBI.

Keywords: Independent component analysis, FOBI, GARCH, SOBI.

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