Subsampling inference for the autocorrelations of GARCH processes

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A self-normalization for the sample autocorrelations of GARCH$(1,1)$ process (and of the square-/absolute-value processes) is provided. In the asymptotic distribution of the sample autocorrelations of GARCH$(1,1)$ there are three rates of convergence that depend on the index of the regular variation of GARCH$(1,1)$ process, $\kappa > 0$. Consequently, there are three different limit distributions: when $k > 4$ the limit is Gaussian with the standard $\sqrt{n}$-rate, but the asymptotic var-cov matrix is given by the generalized, instead of the standard, Bartlett formula; when $\kappa \in (2,4)$ the limit is stable, κ-dependent, with nonanalytic cdf, and the rate is roughly $n^{1-2/\kappa}$; when $\kappa < 2$, the sample autocorrelations converge in distribution. We introduce a self-normalized sample autocorrelation statistic, which is computable without knowledge of κ (in particular, we need not assume the finiteness of the fourth moment), and which converges to a nondegenerate distribution. The sampling distribution can then be approximated nonparametrically by subsampling, as the corresponding asymptotic distribution is still parameter-dependent. The subsampling-based confidence intervals for the process autocorrelations are shown to have satisfactory empirical coverage rates in a simulation study. The impact of subsampling block size on the coverage is assessed. The methodology is further applied to daily returns of CAC40 and FTSA100 indices, their squares and absolute values.

Keywords: GARCH processes, parameter-dependent convergence rates, self-normalization, subsampling confidence intervals, heavy-tails, convergence of point-processes.

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