Tests for regression quantiles

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Let $Y_1, \ldots, Y_n$ be observations following the regression model

$$Y_i = \mathbf{x}_i^T \boldsymbol{\beta} + u_i, \qquad i = 1, \ldots, n$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ip}) \in R^p$, $x_{i1} \equiv 1$, $\boldsymbol{\beta} \in R^p$, and $u_i \sim G$ with density g. The regression quantile estimator $\hat{\boldsymbol{\beta}}_\alpha$ is the solution of the minimization problem

$$\hat{\boldsymbol{\beta}}_\alpha = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} \rho_\alpha(Y_i - \mathbf{x}_i^T \boldsymbol{\beta}),$$

where

$$\rho_\alpha(u) = |u| \left\{ (1-\alpha)\, \mathrm{I}\,[u < 0] + \alpha\, \mathrm{I}\,[u > 0] \right\}.$$

Consider the simple hypothesis

$$H_0 : \boldsymbol{\beta}_\alpha = \boldsymbol{\beta}_{\alpha 0}.$$

In this contribution, two different tests will be proposed and compared in a numerical study. The first one being a nonparametric saddlepoint test that does not require the estimation of the sparsity function, with the test statistic given by an explicit formula. The second one is based on the distribution of averaged regression quantiles $\frac{1}{n}\sum_{i=1}^{n} \mathbf{x}_i^T \hat{\boldsymbol{\beta}}_\alpha$ and unlike the first test requires estimation of the sparsity function $s(\alpha) = [g(G^{-1}(\alpha))]^{-1}$.

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