Wild Bootstrap Tests for Autocorrelation in Vector Autoregressive Models

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IN this paper we consider tests for error autocorrelation (AC) in vector autoregressive (VAR) models when the errors are conditionally heteroskedastic.

The K-dimensional vector of time series variables $\mathbf{X}_t$ is assumed to be generated by a vector autoregressive (VAR) model of order p:

$$\mathbf{X}_t = \mu + \mathbf{\Pi}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{\Pi}_p \mathbf{X}_{t-p} + \varepsilon_t, \qquad t = 1, \ldots, T. \tag{1}$$

The error process $\{\varepsilon_t\}$ is assumed to have mean zero and covariance matrix $\mathbf{\Omega}$, nonsingular and positive definite. The Breusch–Godfrey (BG) Lagrange multiplier (LM) test (see e.g. Godfrey 1991) assumes a VAR(h) model for the error terms under the alternative:

$$\varepsilon_t = \mathbf{\Psi}_1 \varepsilon_{t-1} + \cdots + \mathbf{\Psi}_h \varepsilon_{t-h} + \mathbf{e}_t. \tag{2}$$

The LM statistic can be computed from an auxiliary regression

$$\widehat{\varepsilon}_t = \mu + (\mathbf{Z}_t' \otimes \mathbf{I}_K)\phi + (\widehat{\mathbf{E}}_t' \otimes \mathbf{I}_K)\psi + \mathbf{e}_t, \tag{3}$$

where $\mathbf{Z}_t' = (\mathbf{X}_{t-1}', \ldots, \mathbf{X}_{t-p}')$, $\phi = \mathrm{vec}(\mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_p)'$, $\widehat{\mathbf{E}}_t' = (\widehat{\varepsilon}_{t-1}', \ldots, \widehat{\varepsilon}_{t-h}')$ and $\psi = \mathrm{vec}(\mathbf{\Psi}_1, \ldots, \mathbf{\Psi}_h)'$. The symbol $\otimes$ denotes the Kronecker product and the symbol vec denotes the column vectorisation operator. The LM statistic is given by

$$Q_{LM} = T\widehat{\psi}'(\widehat{\mathbf{\Sigma}}^{\psi\psi})^{-1}\widehat{\psi}, \tag{4}$$

where $\widehat{\psi}$ is the generalised least squares (GLS) estimate of ψ from the auxiliary regression, $\widehat{\mathbf{\Sigma}}^{\psi\psi}$ is the block of

$$\left(T^{-1} \sum_{t=1}^{T} \begin{bmatrix} \widehat{\mathbf{E}}_t \otimes \mathbf{I}_K \\ \mathbf{Z}_t \otimes \mathbf{I}_K \end{bmatrix} \widehat{\mathbf{\Omega}}^{-1} \begin{bmatrix} \widehat{\mathbf{E}}_t' \otimes \mathbf{I}_K & \mathbf{Z}_t' \otimes \mathbf{I}_K \end{bmatrix} \right)^{-1} \tag{5}$$

corresponding to ψ and $\widehat{\mathbf{\Omega}} = T^{-1} \sum_{t=1}^{T} \varepsilon_t \varepsilon_t'$.

The asymptotic Q_{LM} test is invalid if the errors are conditionally heteroskedastic. Tests based on heteroskedasticity-consistent covariance matrix estimators (HCCME) are asymptotically valid. We show in simulation experiments that asymptotic tests are oversized, whereas asymptotic tests based on the HCCME can be seriously undersized and have low power in the presence of strong persistence in volatility in the form of ARCH errors. Wild bootstrap

(WB) tests perform well both when the errors are independent and identically distributed (IID) and conditionally heteroskedastic. WB tests based on the HCCME have the smallest error in rejection probability in finite samples. Another result of practical importance is that WB tests based on the HCCME have much lower power than WB tests without the HCCME.

Table 1 shows the size and Figure 1 the simulated power functions for $K = 2$, $T = 100$, $h = 1$, 4 and 12, and constant conditional correlation generalised autoregressive conditional heteroskedasticity (CCC-GARCH) errors with ARCH parameters 0.08, GARCH parameters 0.9 and conditional correlation coefficient 0.5.

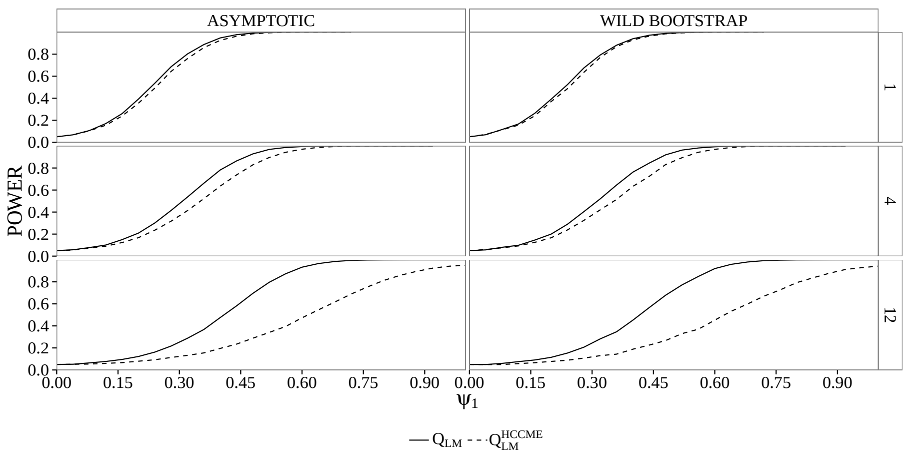

Figure 1: Simulated power functions of the Q_{LM} tests for error AC

Table 1: Size of asymptotic and WB Q_{LM} tests for error autocorrelation

h	Q_{LM}	Q_{LM}^{HCCME}	Q_{LM}^{WB}	$Q_{LM}^{HCCME,WB}$
1	0.074	0.034	0.054	0.051
4	0.082	0.013	0.053	0.051
12	0.061	0.000	0.055	0.052

Keywords: Wild bootstrap, Vector autoregressive model, Conditional heteroskedasticity, Error autocorrelation.

References:

Godfrey, L. G., (1991). *Misspecification Tests in Econometrics.* Cambridge: Cambridge University Press.