Exit times for autoregressive processes with different distributions

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Consider the process

$$X_t = aX_{t-1} + \varepsilon\xi_t, t \geq 1, X_0 = 0, \tag{1}$$

where $|a| < 1$, ε is a positive parameter and $\{\xi_t\}_{t\geq 1}$ is a sequence of independent and identically distributed random variables. Let τ be the time until the process exits from $(-1, 1)$, that is, $\tau := \min\{t \geq 1 : |X_t| \geq 1\}$. When ξ_1 has a standard normal distribution, it has been shown that

$$\lim_{\varepsilon \to 0} q(\varepsilon) \log E\tau = \frac{1}{2}(1 - a^2), \tag{2}$$

for $q(\varepsilon) = \varepsilon^2$. We now study such limits for other distributions than the Gaussian one. The function $q(\varepsilon)$ depends on the choice of distribution. We see that the value of the limit does not always depend on a. As an example, we show that

$$\lim_{\varepsilon \to 0} \varepsilon \log E\tau = \frac{1}{b}, \tag{3}$$

when ξ_1 have a Laplace distribution with location parameter 0 and scale parameter b (the mean of ξ_1 is 0 and the variance is $2b^2$).

Keywords: autoregressive processes, exit time, Laplace distribution

References:

Jung, B., (2013). Exit times for multivariate autoregressive processes, *Stoch. Proc. Appl. 123*, 8, 3052–3063.