

# LIKELIHOOD ANALYSIS OF THE AGE-PERIOD-COHORT MODEL

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The age-period-cohort model is extensively used in actuarial sciences, demography, epidemiology and social sciences. It is surrounded with controversy because of its inbuilt identification issue, see for instance Luo (2013). The issue is that the predictor of the model is set up as a linear combination of time effects for age, period and cohort, but these time effects cannot be fully recovered from the predictor. Kuang, Nielsen and Nielsen (2008) analysed the problem and argued that the time effects should serve as a motivation for a model, but that they are not in themselves objects of interest. Rather, attention should focus on the mathematical structure of the predictor. This results in a parsimonious, freely varying parametrisation which reveals which questions that can be addressed by the age-period-cohort model. This paper provides two extensions and is accompanied by an R package.

First, the role of the data array is analysed. Data is usually available as one of the three Lexis diagram. These are rectangular data arrays with age-period, age-cohort or period-cohort coordinates. A unified parsimonious parametrisation is suggested.

Secondly, a set of 14 sub-models, such as the age-cohort model are formulated. The age-period-cohort model and these 14 sub-models are easily estimated within a generalized linear model framework. Collecting the likelihood values in an anova-type table gives a convenient overview of the sub-models.

The method is illustrated using the Belgian lung cancer data from Clayton and Schifflers (1987) and the UK mesothelioma mortality data from Martínez Miranda, Nielsen and Nielsen (2014).

**Keywords:** Age-period-cohort model, canonical parametrisation, identification, generalized linear model.

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