Invariant Coordinate Selection revisited: Fisher Symmetry and Symmetric Component Analysis

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Tyler et al. (2009) introduced invariant coordinate selection, or ICS, as a general method for exploring affine invariant features of multivariate data by comparing different estimates of multivariate scatter. Together with Critchley et al. (2006), they report examples of the method performing well for a wide range of problems, extending beyond the limits of existing theoretical support. Motivated by this, we provide complementary ICS theory based on the relevant symmetry group. A *Fisher symmetry* condition is introduced for which elliptical symmetry is not required, yet under which a subset of the invariant coordinates is shown to correspond to Fisher's linear discriminant subspace, class identifications of data points remaining unknown. Again, a *Symmetric Component Analysis* model is introduced in which independence is not required, yet under which the invariant coordinates are seen to correspond to the symmetric components. Illustrative examples are given. Further developments are briefly indicated.

Keywords: Fisher's linear discriminant subspace, invariant coordinate selection (ICS), symmetric component analysis (SCA)

References:

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