

ON EXPECTED DURATIONS FOR BRANCHING AND OTHER BIRTH-DEATH TYPE PROCESSES

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We study continuous-time birth-death processes, where individuals have i.i.d. life-time distributions Q , subject to $E(Q) = 1$, and where the birth rate if currently in state k is λ_k . We focus on two examples, namely $\lambda_k = \lambda k$ being a branching process, and $\lambda_k = \lambda k(N - k)$ which corresponds to an SIS epidemic model in a community of fixed size N . The processes are assumed to start in state 1. Let T and $T^{(N)}$ respectively denote the (random) time to extinction for the branching process and the SIS-epidemic, and let T_k and $T_k^{(N)}$ denote the time spent in state k for the two processes. The main result of the paper states what the expectation of all these expressions are, and showing that these expectations are insensitive to the distribution of Q . We also derive an expression for the expected time to extinction of the SIS-epidemic, but now starting at the endemic state; an expression which is *not* independent of the distribution of Q .

Keywords: Branching process, SIS-epidemic, extinction time, insensitivity