Optimal stopping of strong Markov processes on the real line

Sören Christensen[1 2], **Paavo Salminen**[3], **and Bao Ta Quoc**[3]

[1] Hamburg University, Germany

[2] Christian-Albrechts-University, Germany

[3] Abo Akademi, Finland

We characterize the value function and the optimal stopping time for a large class of optimal stopping problems of the form

$$V(x) := \sup_{\tau} \mathbb{E}_x(e^{-\beta\tau} G(X_\tau)) = \mathbb{E}_x(e^{-\beta\tau^*} G(X_{\tau^*})),$$

where the underlying process X to be stopped is a fairly general Markov process.

For processes on the real line, our main result is inspired by recent findings for Lévy processes obtained essentially via the Wiener-Hopf factorization. The main ingredient in our approach for this more general situation is the representation of β-excessive functions u as expected suprema $u(x) = \mathbb{E}_x(\sup_{0\leq t\leq T} f(X_t))$ for an independent $Exp(\beta)$-distributed time T. Under quite general assumptions, we show that the value function can be written in the form

$$V(x) = \mathbb{E}_x\left(\sup_{0\leq t\leq T} \hat{f}(X_t) 1_{\{X_t\geq x^*\}}\right) = \mathbb{E}_x\left(\hat{f}(M_T) 1_{\{M_T\geq x^*\}}\right)$$

for some function $\hat{f}$, where M denotes the running maximum process, and

$$\tau^* := \inf\{t \geq 0 : X_t > x^*\}$$

is an optimal stopping time. We describe an approach how to find the function $\hat{f}$ and the stopping threshold x^* explicitly and discuss it for a variety of examples.

Keywords: Optimal stopping problem; Markov processes; Hunt processes; Lévy processes; Supremum representation for excessive functions.

References:

Christensen, S., P. Salminen, B. Ta Quoc (2013). Optimal stopping of strong Markov processes *Stochastic Processes and their Applications 123*, 1138–1159.

Novikov, A, A. Shiryaev (2007). On solution of the optimal stopping problem for processes with independent increments *Stochastics*, *79*, 393–406.

Surya, B. A. (2007). An approach for solving perpetual optimal stopping problems driven by Lévy processes *Stochastics*, *79*, 337–361.