Integral representations with respect to Hölder processes

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A natural question in stochastic analysis is to study integral representations of random variables with respect to some given process, i.e. which random variables ξ can be represented as an integral of form

$$\xi = \int_0^T \phi(s) dX_s, \quad a.s. \tag{1}$$

for some adapted integrand $\phi(s)$ and a given process $X = (X_t)_{t\in[0,T]}$. While such representations have also theoretical interest, it is particularly interesting in applications such as mathematical finance. Indeed, ξ can be viewed as the claim and the representation (1) corresponds to the hedging equation. However, compared to classical hedging equations there is a constant C corresponding to the arbitrage free value missing in equation (1). This implies that claims ξ with representation (1) can be hedged without any cost, and this usually leads to arbitrage opportunities.

Classical result by Dudley on such representation states that any $\mathcal{F}_1$-measurable random variable have representation (1) with respect to standard Brownian motion W, where the filtration is generated by Brownian motion W. Note however, that the resulting process $\phi(s)$ does not belong to the class of so-called allowed strategy in the classical Black-Scholes model. Rather surprisingly, extensions of such results was not developed until 2013 by Mishura et al. who studied the case of fractional Brownian motion (fBm) with Hurst index $H > \frac{1}{2}$. The authors considered pathwise integrals and proved that representation (1) holds for wide class of variables ξ. As a consequence of this result it was proved that a financial model driven by geometric fBm admits strong arbitrage with relatively simple integrands which in turn indicates that geometric fBm is not a suitable model for the stock price. Moreover, later these result was extended by Viitasaari to cover a wide class of Gaussian processes. Namely, the Gaussian processes under consideration was assumed to be Hölder continuous of some order $\alpha > \frac{1}{2}$ together with some minor technical assumptions on the covariance. Moreover, it was proved that actually the replication can be done in arbitrary small amount of time which again indicates that such processes are not proper models in finance.

This talk aims to extend such results to cover a wide class of processes. In particular, we prove that the only required facts are Hölder continuity of the process with some index $\alpha > \frac{1}{2}$ together with a mild estimate for small ball probability. As such, the minor technical assumptions from Viitasaari can be dropped. More importantly, the assumption of the Gaussianity can be dropped which is a huge extension to the class of processes considered. The results are based on two papers by Shevchenko and Viitasaari, where the authors have constructed continuous adapted process $\phi(s)$ such that representation (1) holds. The benefit of continuous integrand is that it allows to drop assumptions such as Gaussianity while the results by Mishura et al. and Viitasaari rely on certain Itô formula known to hold only for certain class of Gaussian processes. In this talk we present the results and compare them to previous ones. Applications are discussed.

Keywords: Integral representation, Hölder-continuous processes

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